Results for option pricing

- [o,v,b]=optimal(rand(1,100000))
- Estimators = 0.4619 0.4617 0.4618 0.4613 0.4619
- o = 0.46151 % best linear combination (true value=0.46150)
- v = 1.1183e-005 %variance per uniform input
- b’ = -0.5503 1.4487 0.1000 0.0491 -0.0475

Efficiency of Optimal Linear Combination

- Efficiency gain based on number of uniform random numbers 0.4467/0.00001118 or about 40,000.
- However, one uniform generates 5 estimators requiring 10 function evaluations.
- Efficiency based on function evaluations approx 4,000
- *A simulation using 500,000 uniform random numbers ; 13 seconds on Pentium IV (2.4 Ghz) equivalent to* ***twenty billion*** *simulations by crude Monte Carlo.*

Interpreting the coefficients *b*. Dropping estimators.

- Variance of the mean of 100,000 is 1.18×10^{-10} Standard error is around .00001
- Some weights are negative, (e.g. on *Y1*) some more than 1 (on *Y2*), some approximately 0 (could they be dropped? For example if we drop Y_3 then

$$\text{variance increases to about } 1.6 \times 10^{-10}.$$

Tips…

- If you are simulating to generate resuts for a more complicated model (e.g. asian option, non-normal distribution etc) use a simple model (European option, normal distribution etc) as contol variate. Use simulation to estimate the difference (assuming you know the result for the control)
- Allow the uniform variates as input parameters. This facilitates variance reduction without changing the program.
- Try a variety of variance reduction techniques (5-10) including some with second-difference like expressions. Your best estimator is usually an optimal linear combination.
- Only combine antithetic random numbers additively.

Black-Scholes price in R

- blsprice=function(So,strike,r,T,sigma,div){
- d1<-(log(So/strike)+(r-div+(sigma^2)/2)*T)/(sigma*sqrt(T))
- d2<-d1-sigma*sqrt(T)
- call<-So*exp(-div*T)*pnorm(d1)-exp(-r*T)*strike*pnorm(d2)
- put=call-So+strike*exp(-r*T)
- c(call,put)}

Useful URL's

- http://www.std.com/nr/index.html (numerical recipies)
- http://www.cboe.com
- http://rweb.stat.umn.edu/R/ (R library)

Simulating Survivorship bias and the maxima of Brownian Motion

Examples in Biostatistics: Sequential tests for a mean. e.g. For testing hypothesis

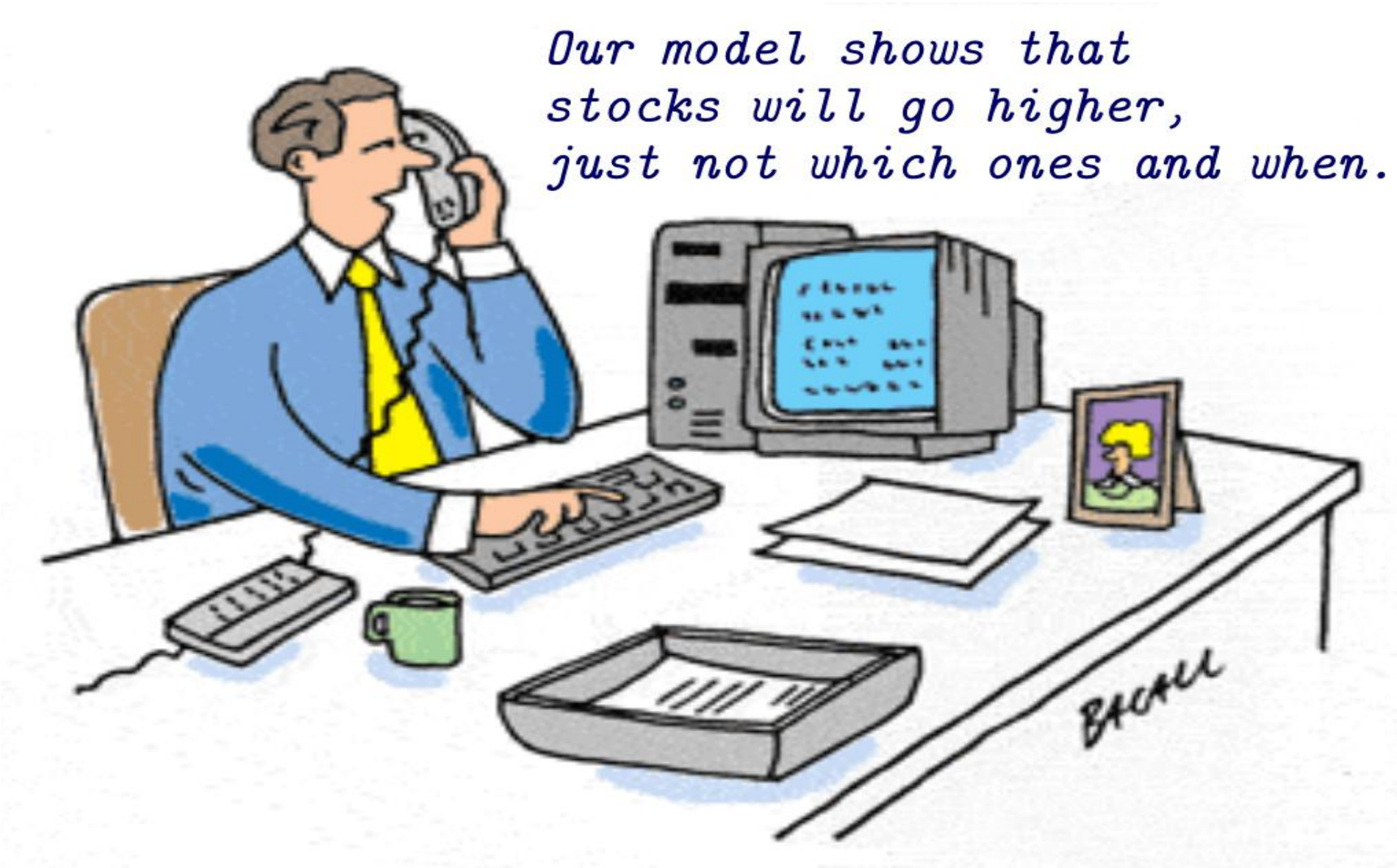
$$\mathrm{H}_0 : \mu \geq 0$$

$$H_1 : \mu < 0$$

$$\text{Reject } \mathrm{H}_0 \text{ as soon as } B(t) < -c_0 - c_1 t.$$

$$i.e. \min\{B(t) + c_0 + c_1 t < 0\}$$

Modeling highs



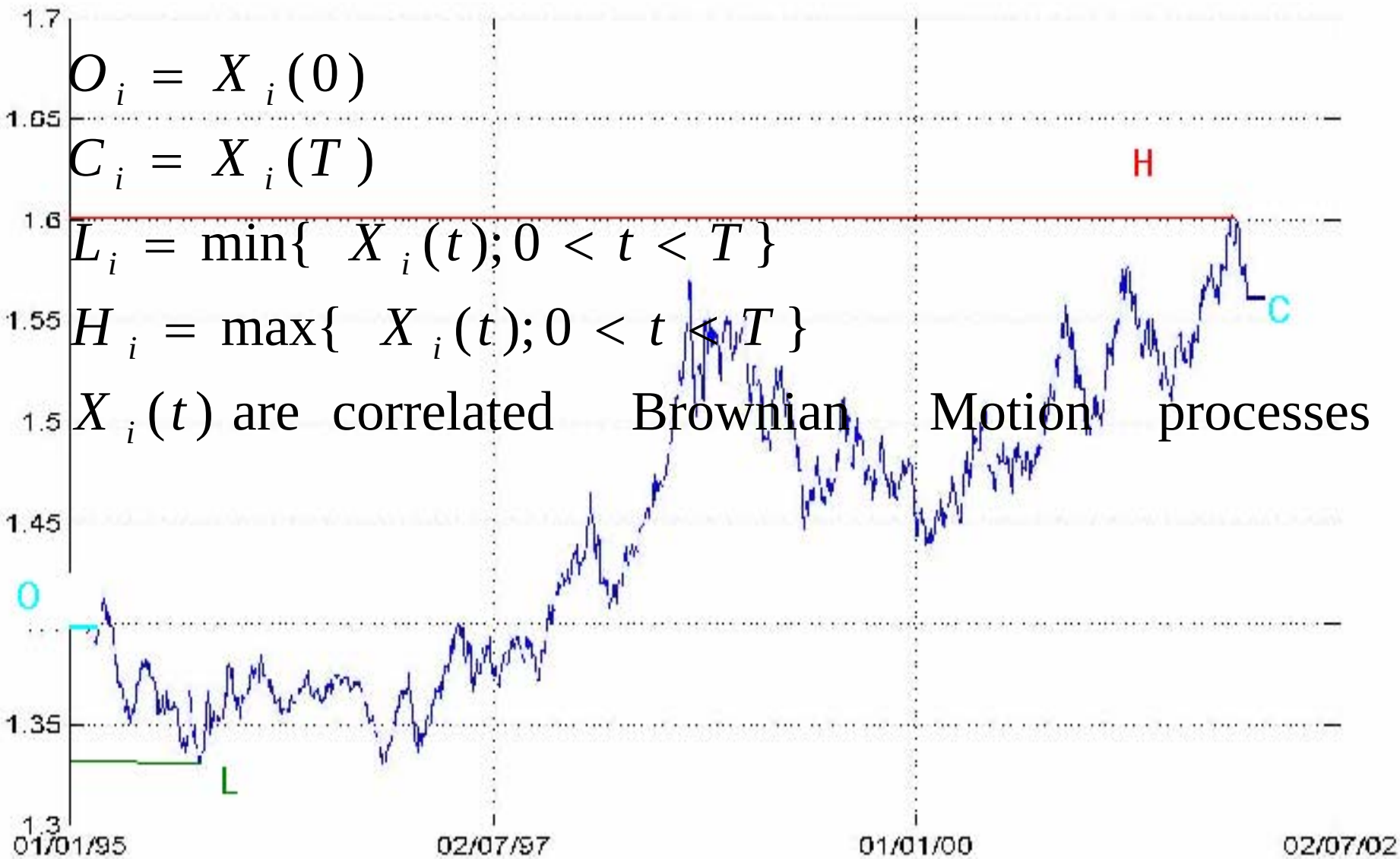
Brownian Motion

- Brownian motion

$$dX(t) = \mu dt + \sigma dW(t)$$

$$\Delta X(t) \sim N(\mu \Delta t, \sigma^2 \Delta t)$$

We typically observe



$$O_i = X_i(0)$$

$$C_i = X_i(T)$$

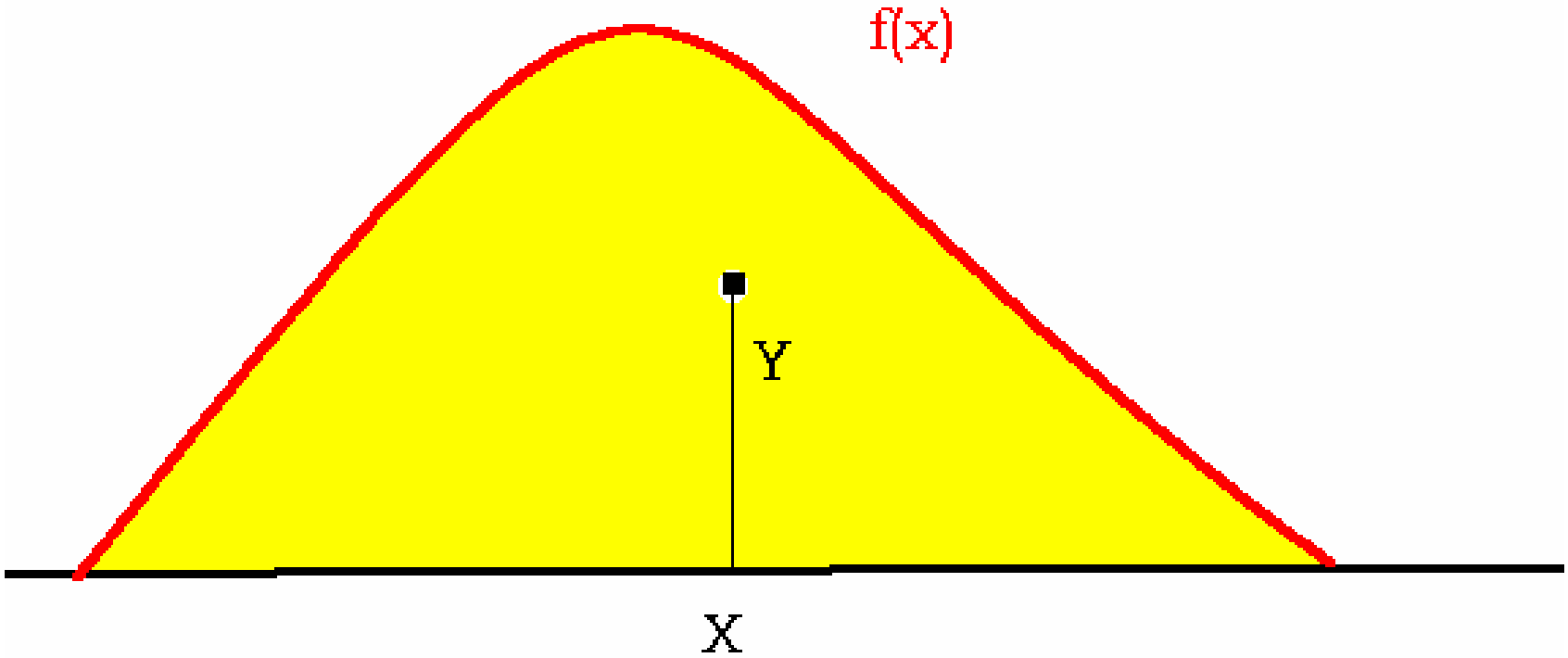
$$L_i = \min\{ X_i(t); 0 < t < T \}$$

$$H_i = \max\{ X_i(t); 0 < t < T \}$$

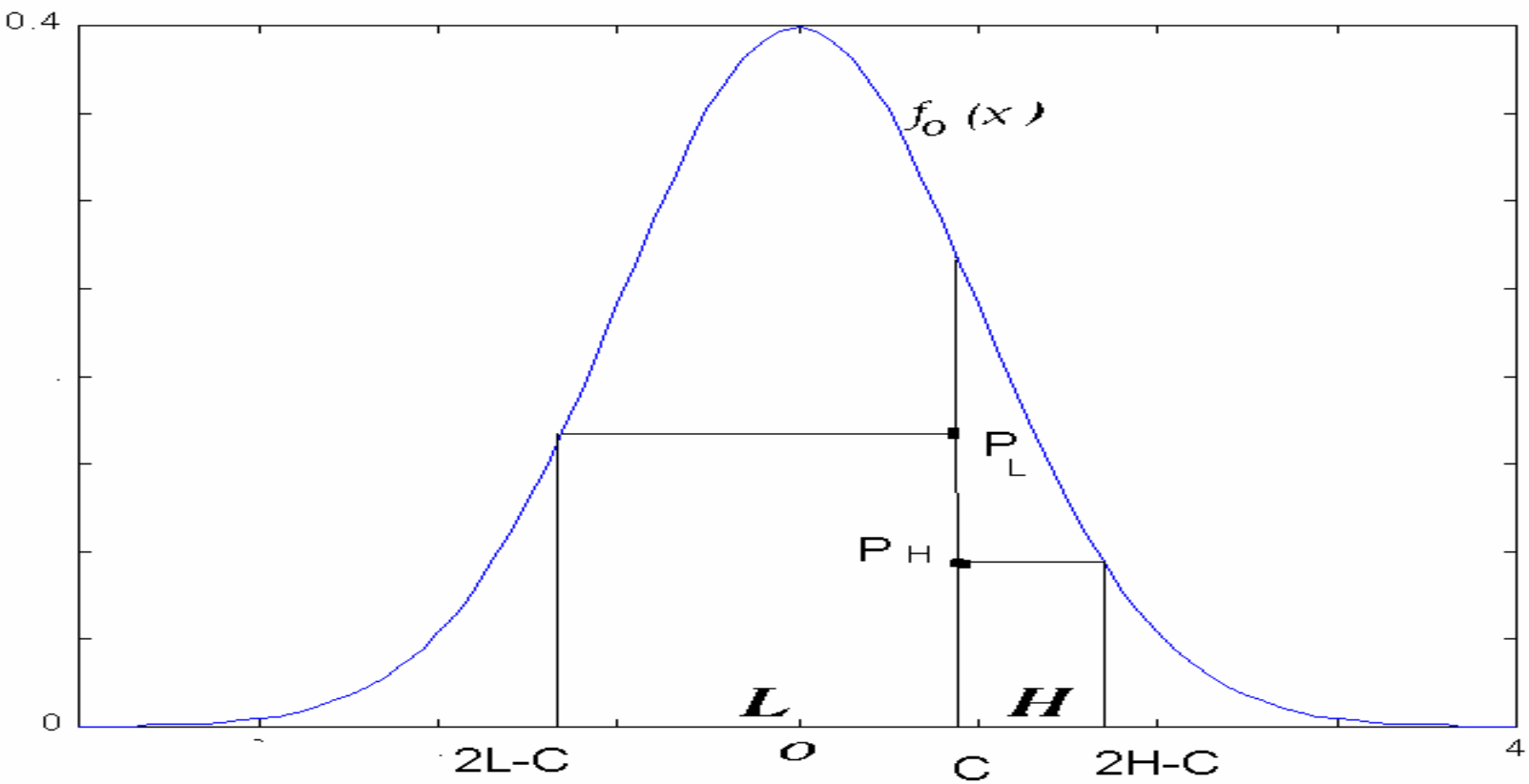
$X_i(t)$ are correlated Brownian Motion processes

Acceptance-rejection generating from f(x)

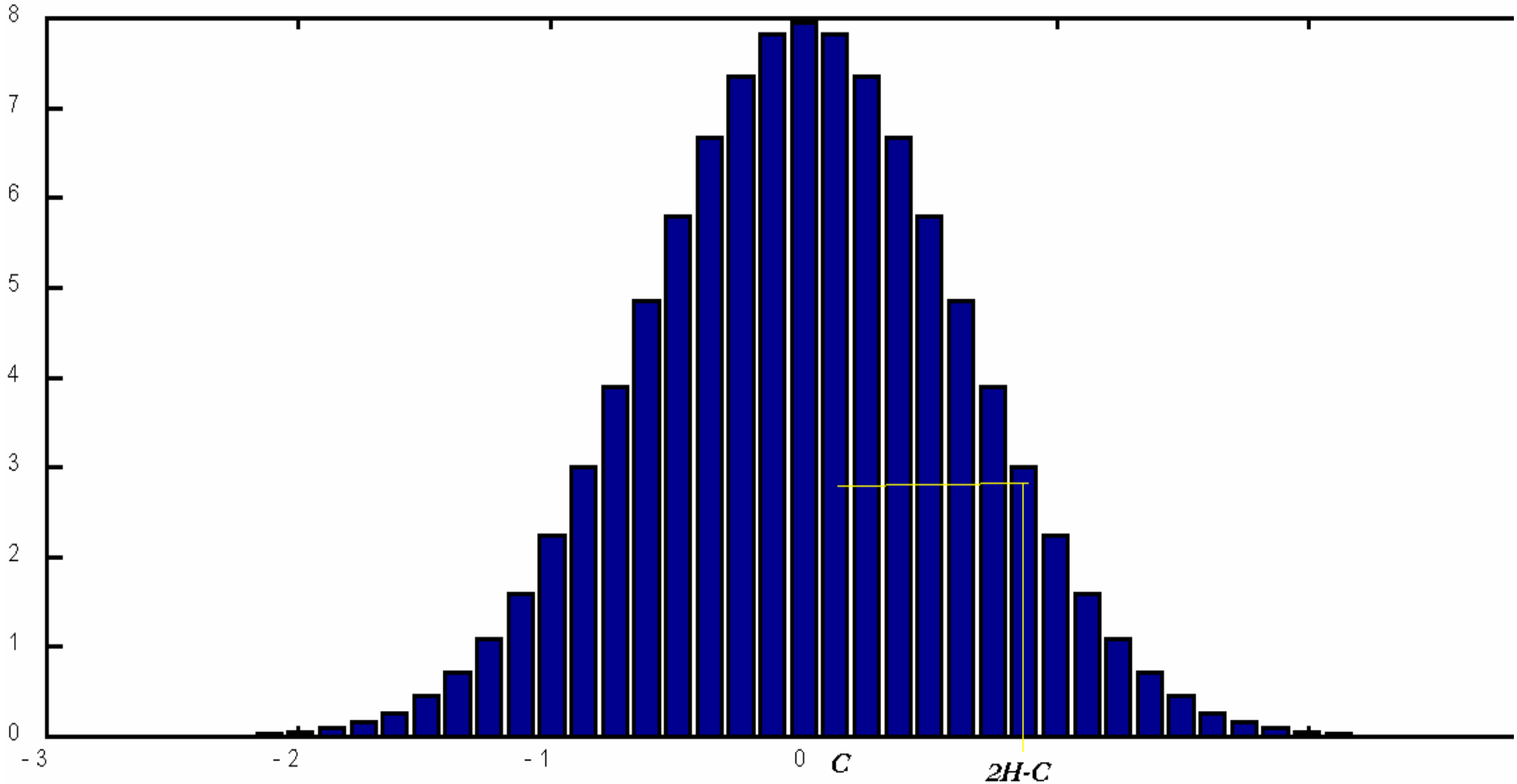
Suppose we generate the Close C by Acceptance-Rejection using its pdf f(x)



We can use the same picture to simulate jointly (H,C) for BM



Similarly if C is discrete



Exponential Statistics

Theorem:

For Brownian Motion,

$$Z_H = (H - O)(H - C) \sim \exp(\sigma^2 T / 2)$$

$$Z_L = (L - O)(L - C) \sim \exp(\sigma^2 T / 2)$$

independent of (O, C).

Proof :

Random Walk and reflection

$$P(H \geq m \mid C = u) = \begin{cases} 1 & m \leq \max(0, u) \\ \dfrac{f(2m-u)}{f(u)} & m > u \end{cases}$$

Substitute observed values in survivor function in Normal case

$U = \dfrac{f(2H - C)}{f(C)}$ is the survivor function for H given C evaluated at its observed values. Therefore it is Uniform[0,1].

But $\mathrm{U} = \dfrac{f(2H - C)}{f(C)} = \exp\{\dfrac{1}{2T\sigma^2}(C^2 - (2H - C)^2)\}$

$= \exp\{-\dfrac{2H(H - C)}{T\sigma^2}\}$

Therefore $-\ln(U) = \dfrac{2H(H - C)}{T\sigma^2}$ is $\exp(1)$

$\mathrm{Z_H} = H(H - C)$ is $\exp(\dfrac{T\sigma^2}{2})$.

Estimating volatility based on High and Low

Both Z_H and Z_L (and average) provide

an estimator of volatility or variance.

Alternative to sample variance of increments

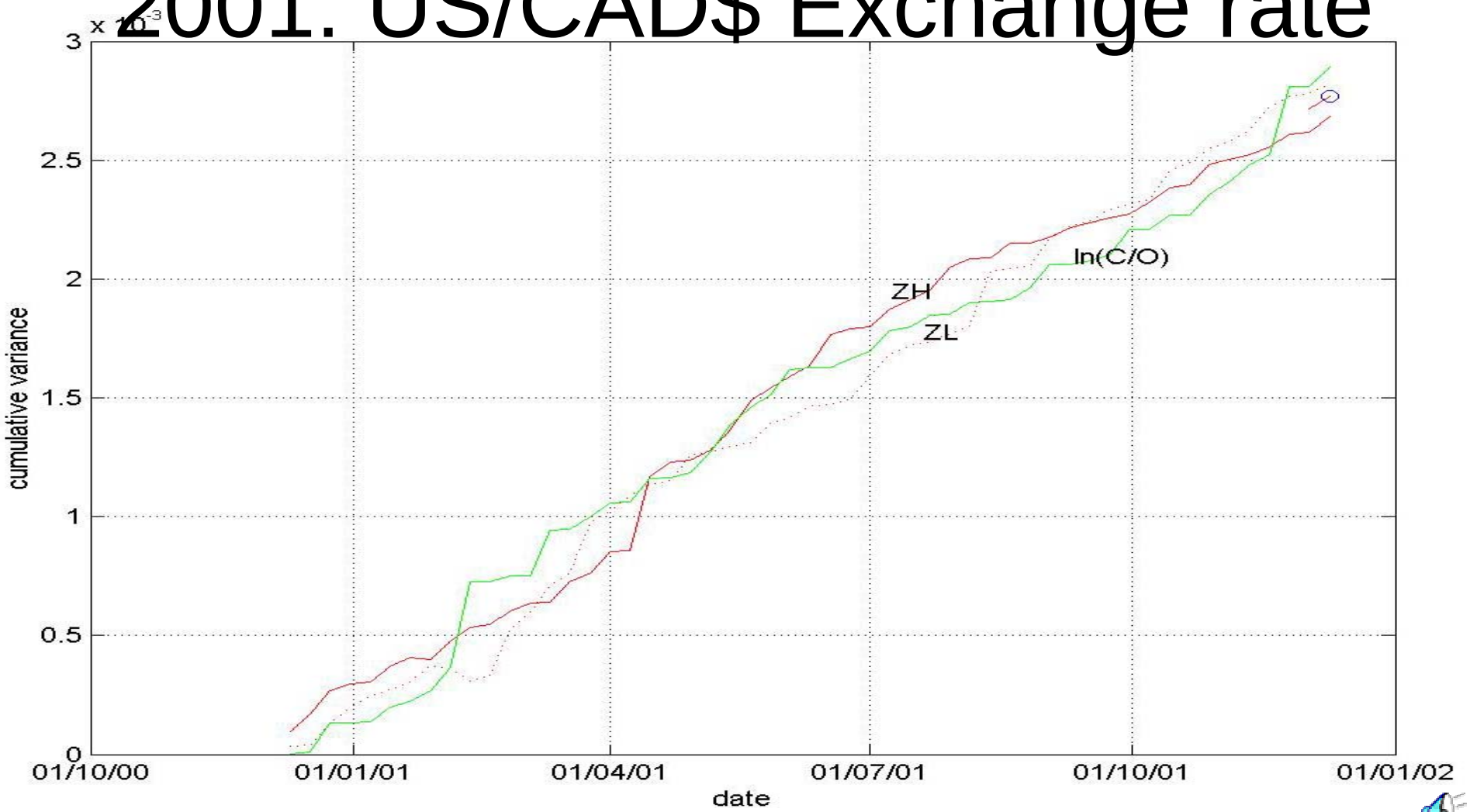
e.g. average$(\mathrm{C}-\mathrm{O})^2$

average$(Z_H + Z_L)$ has about five times the efficiency

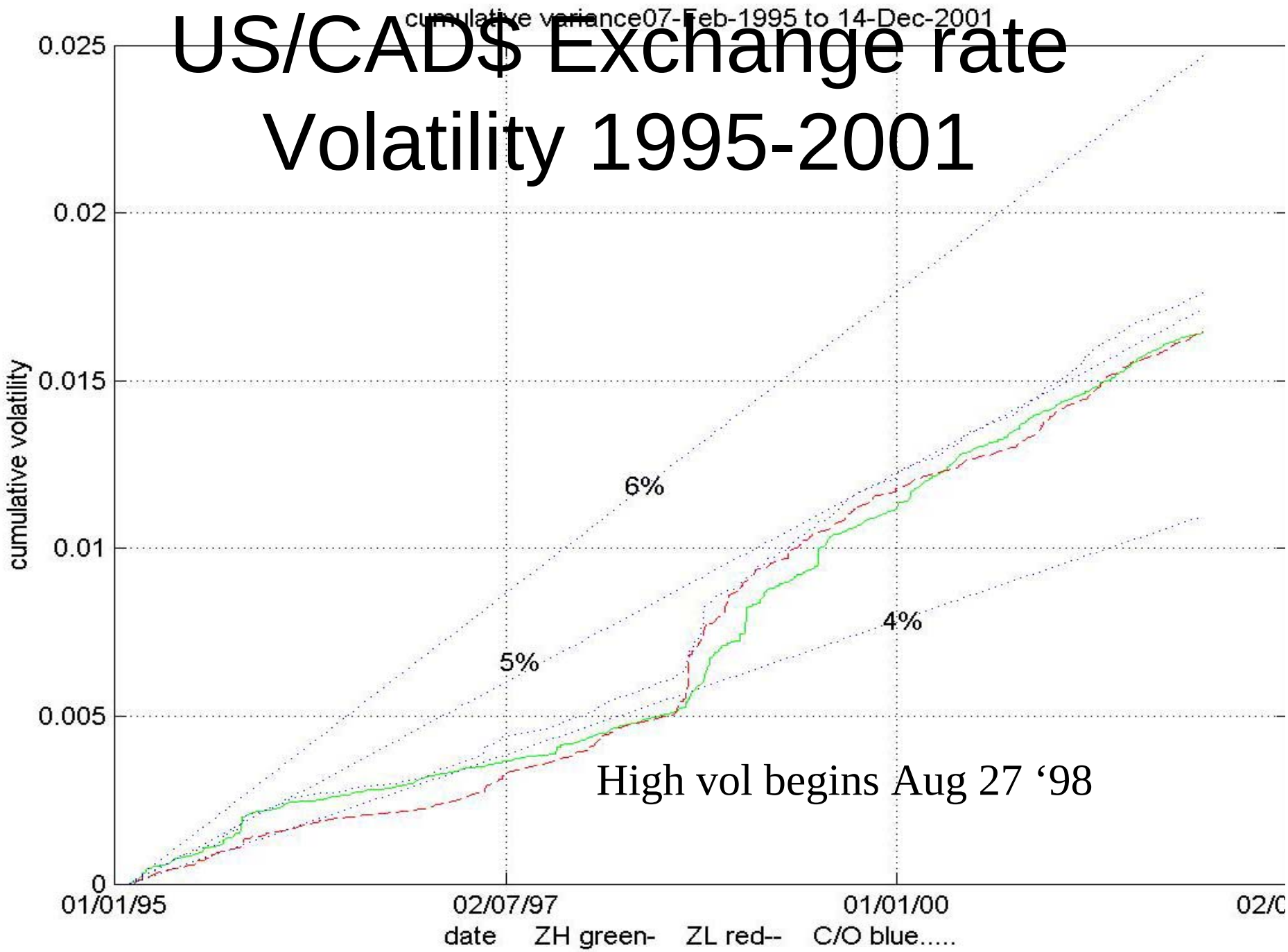
One week high and low equivalent to daily data.

Also unbiased regardless of drift in BM.

Cumulative Variance
2001: US/CAD$ Exchange rate

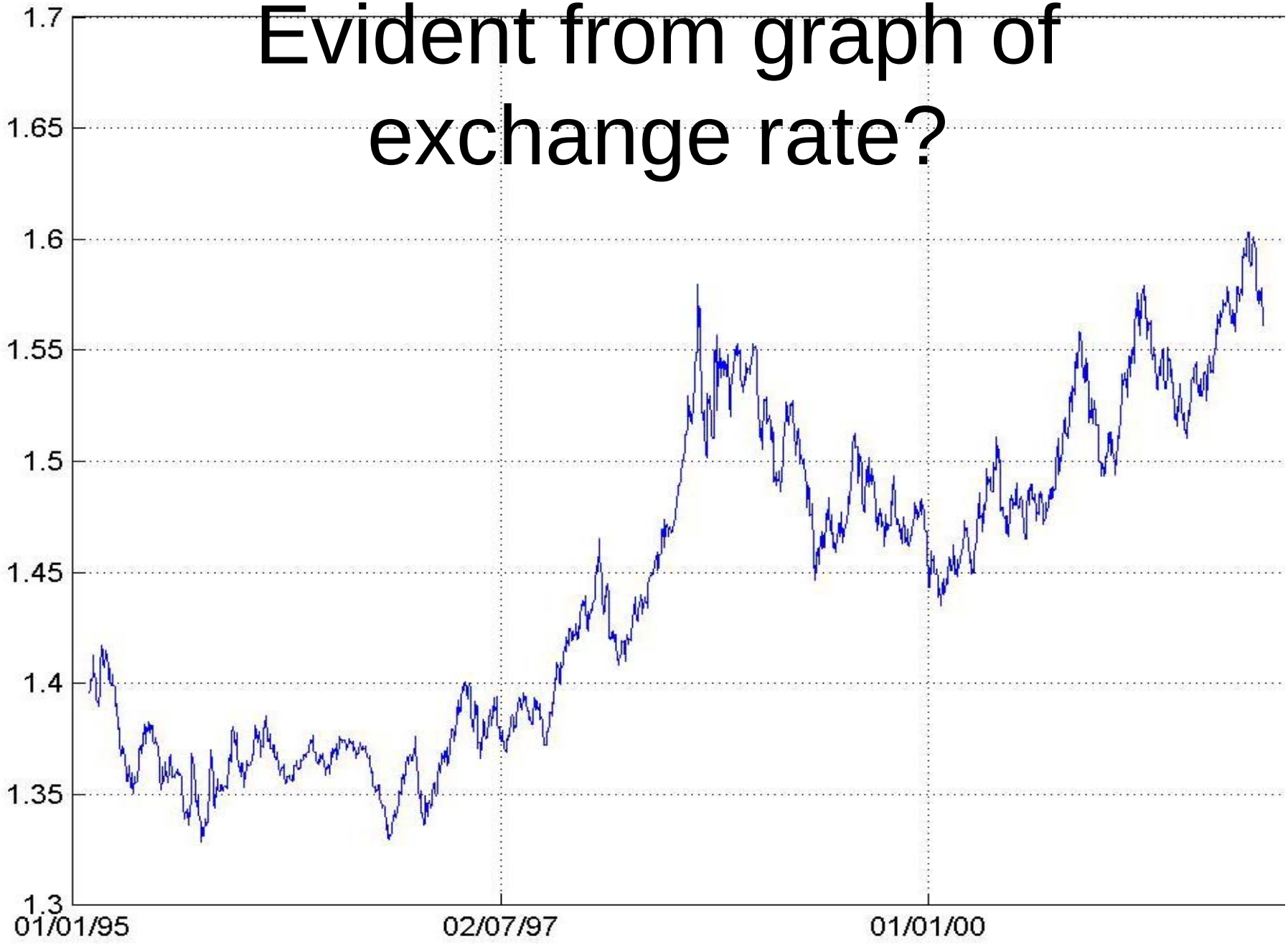


US/CAD$ Exchange rate Volatility 1995-2001

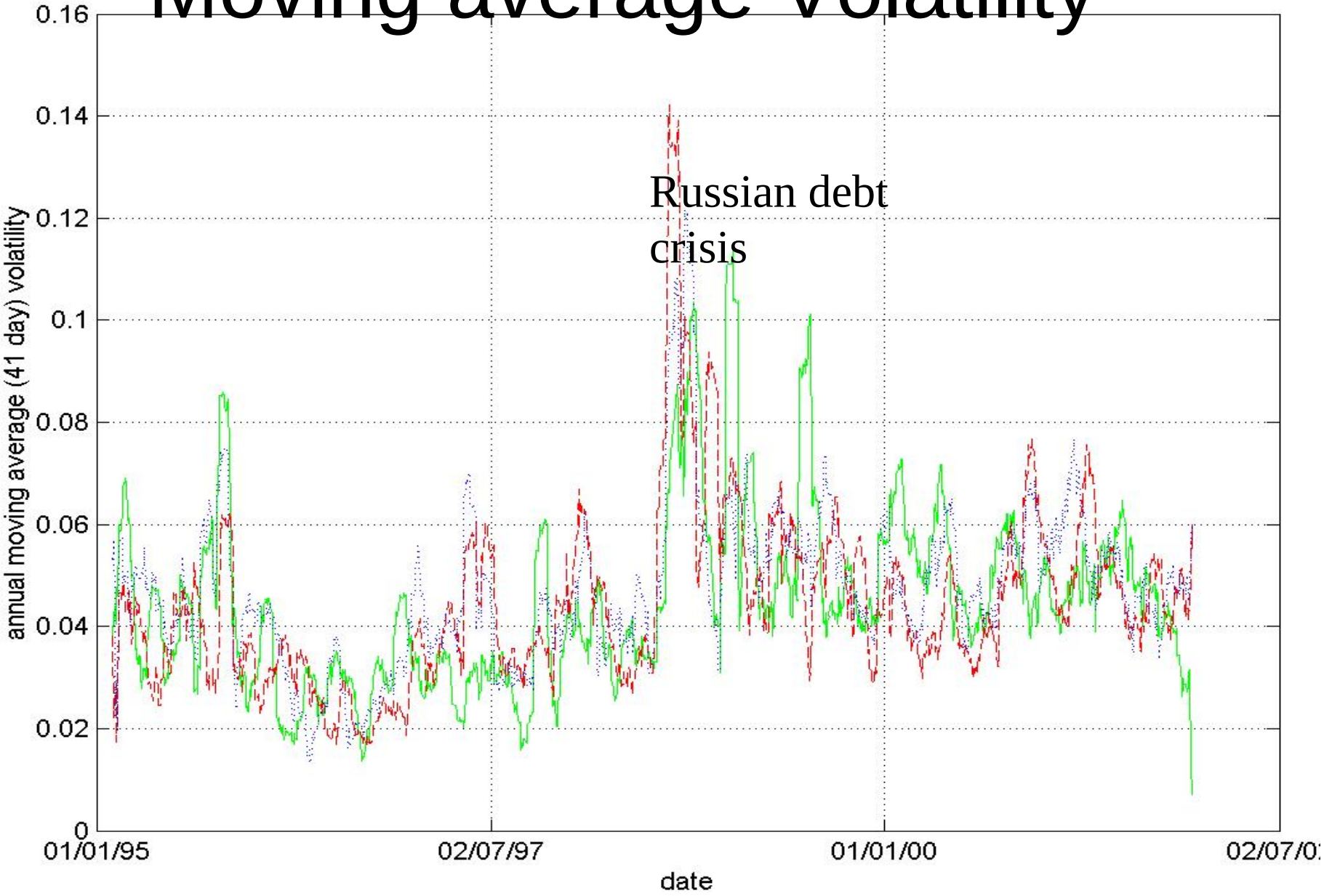


High vol begins Aug 27 ‘98

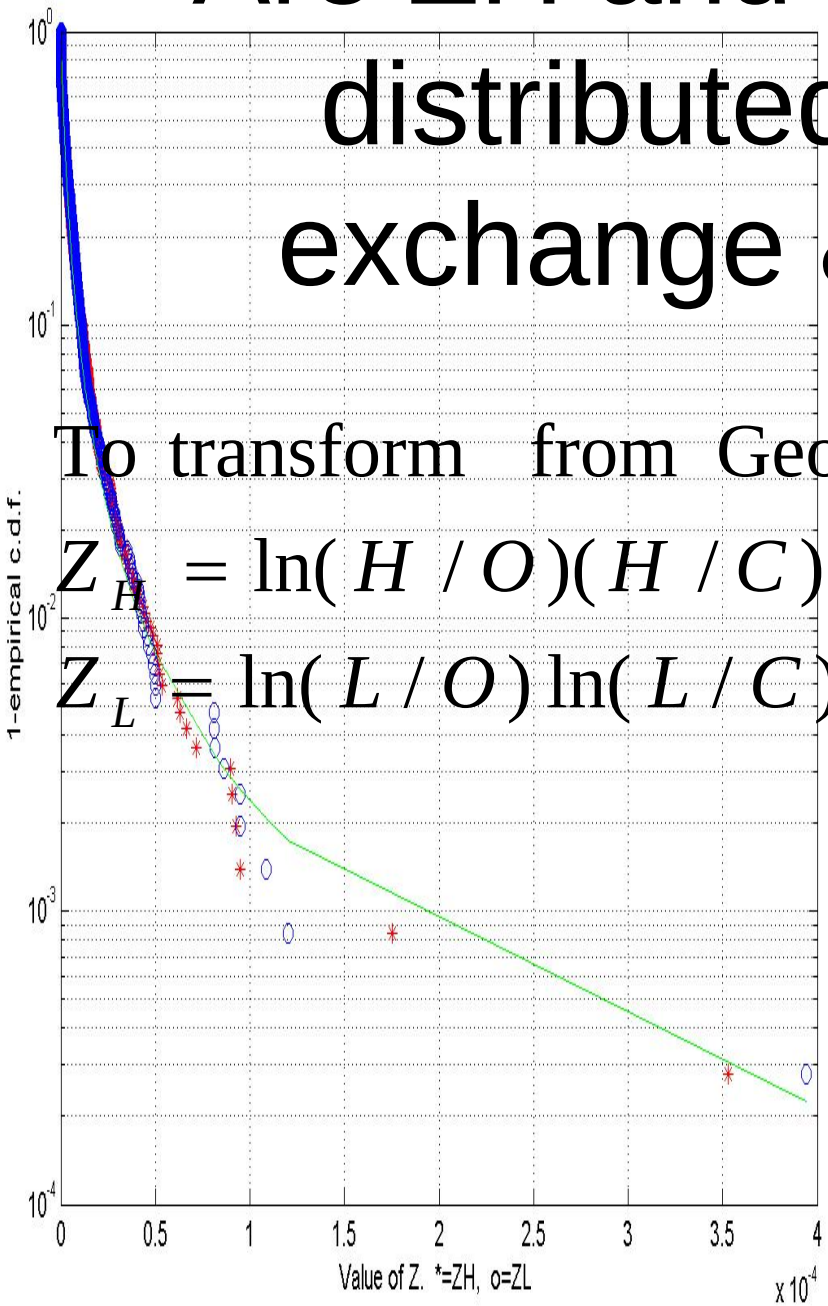
Evident from graph of exchange rate?

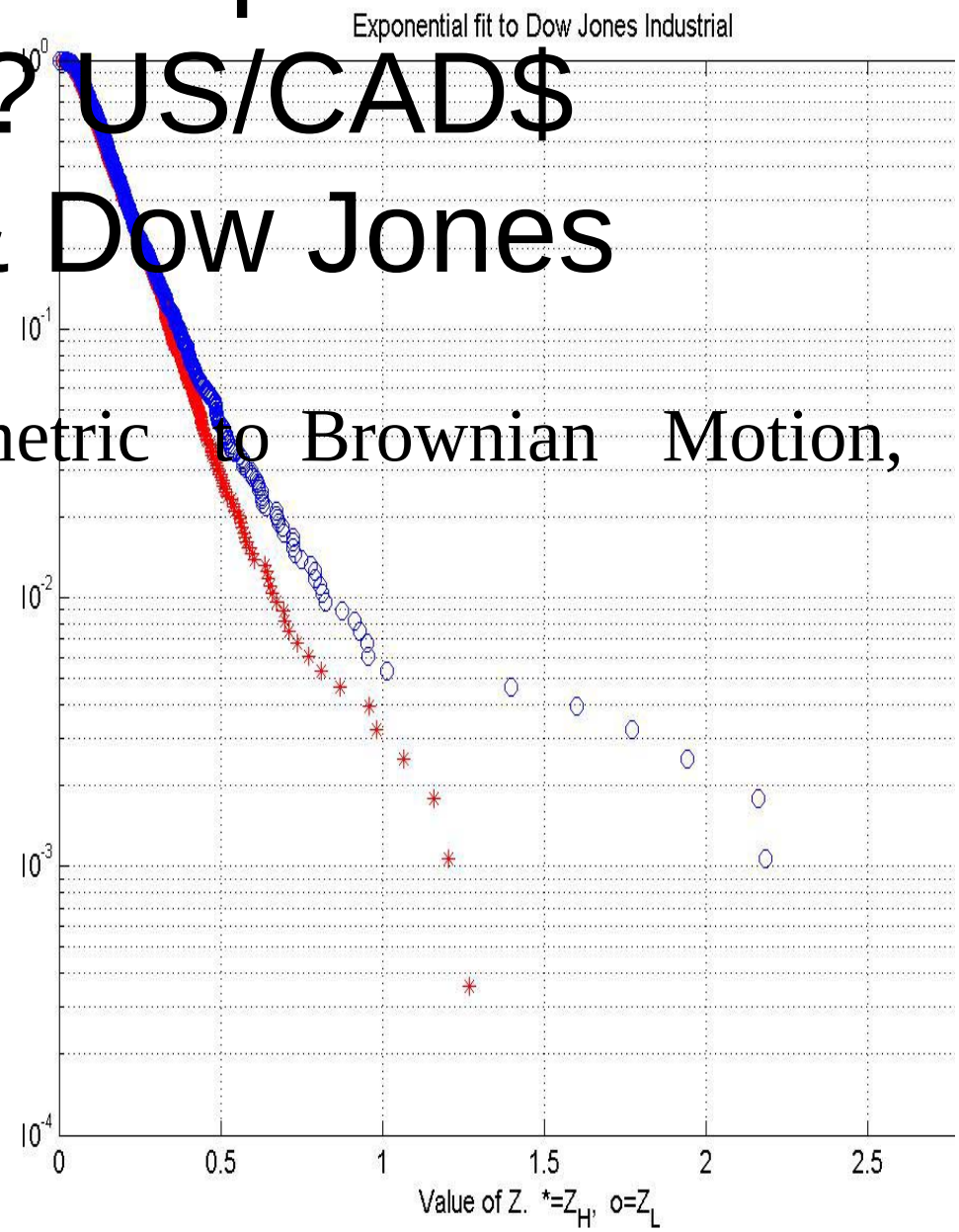


Moving average Volatility



Are ZH and ZL exponential distributed? US/CAD$ exchange & Dow Jones



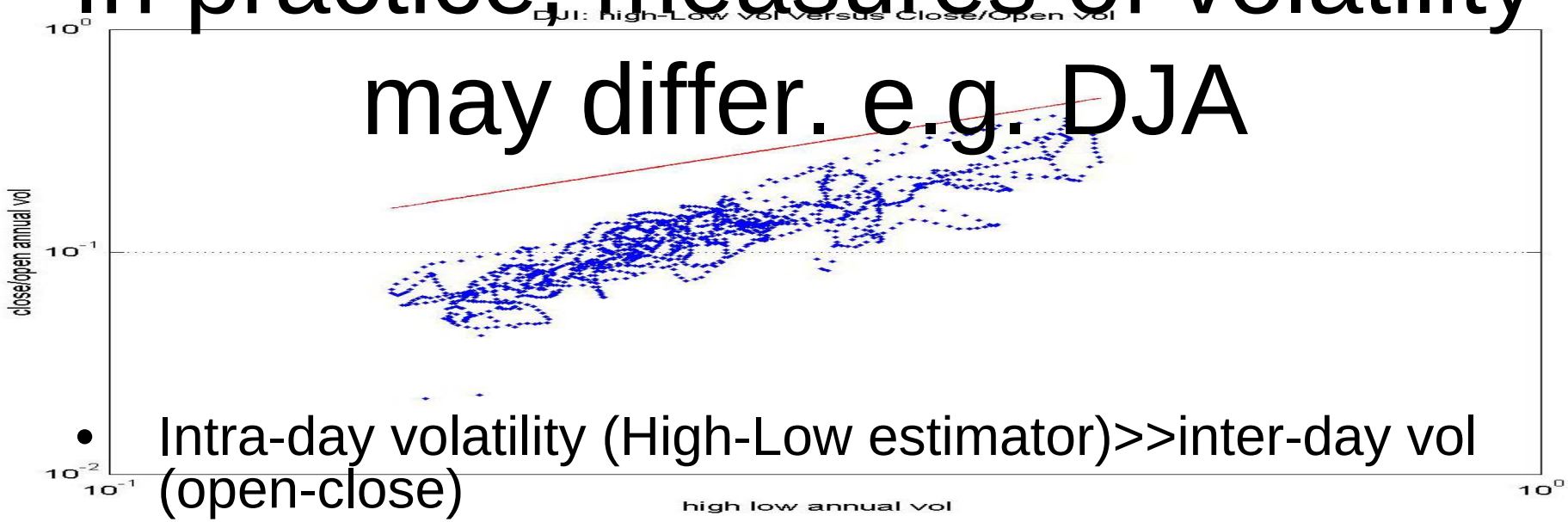


To transform from Geometric to Brownian Motion,

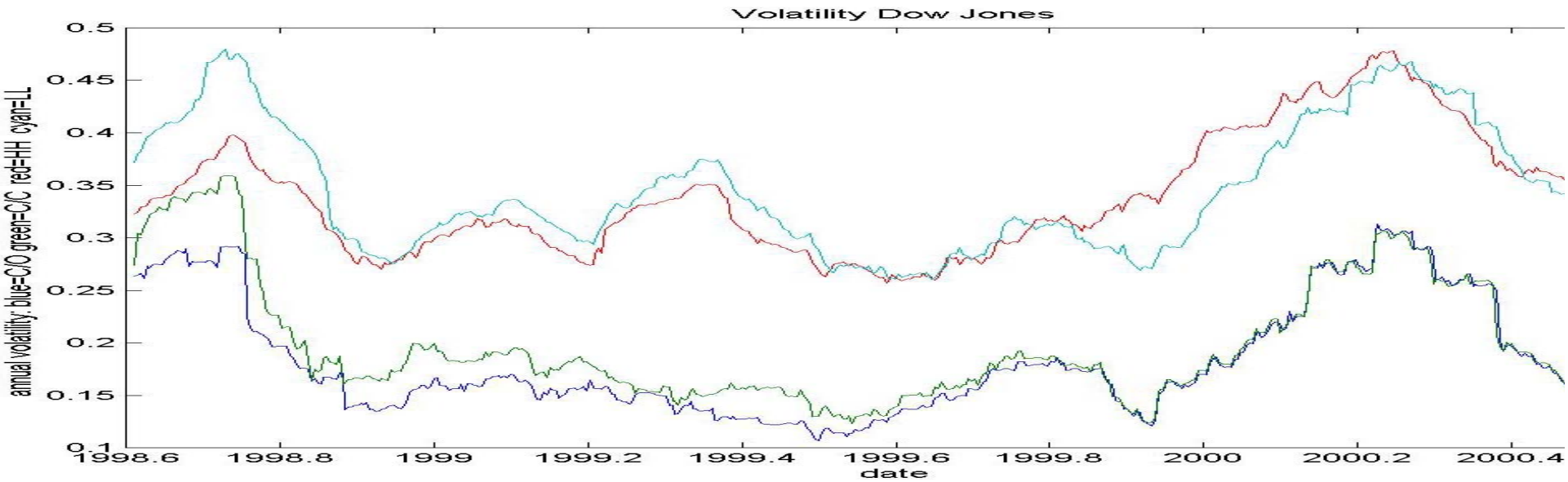
$$Z_H = \ln(H/O)(H/C)$$

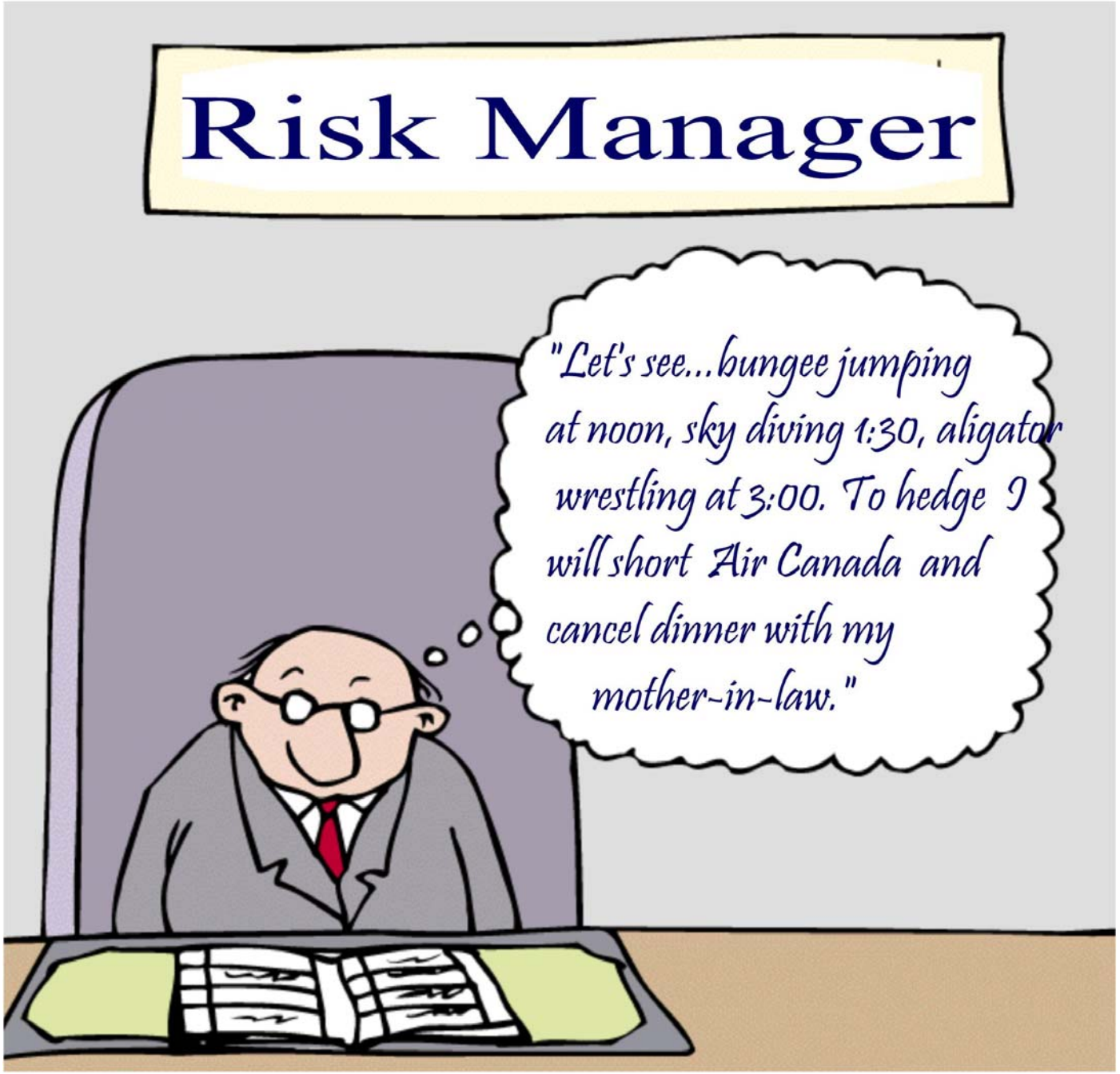
$$Z_L = \ln(L/O)\ln(L/C)$$

In practice, measures of volatility may differ. e.g. DJA



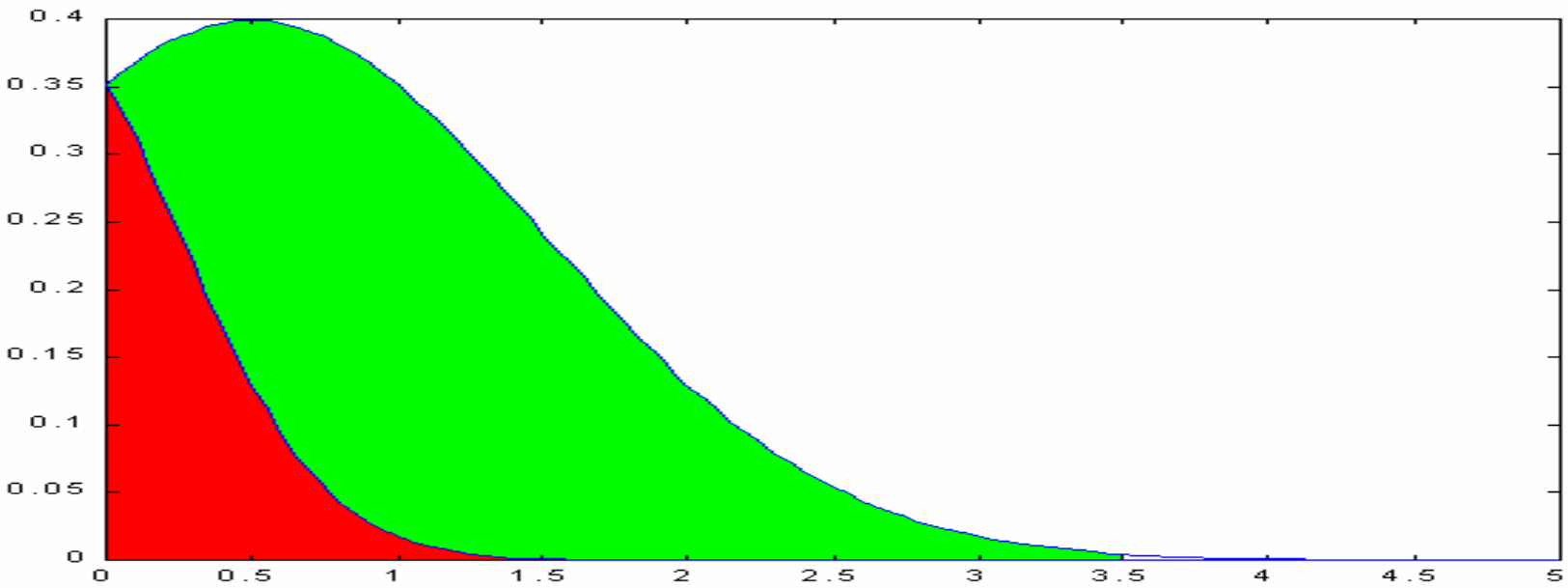
- Intra-day volatility (High-Low estimator)>>inter-day vol (open-close)




Risk Manager
"Let's see...bungee jumping at noon, sky diving 1:30, aligator wrestling at 3:00. To hedge I will short Air Canada and cancel dinner with my mother-in-law."

SURVIVORSHIP BIAS

- E.g. Retrospective studies on performance of mutual fund managers. Present market value is conditional on low >0.



What happens to the mean of the survivors as the variance increases?

- survivemovie