Midterm Exam, Stat 901
November 2001. 90 minutes
Instructor: D. L. McLeish

Do any Four (4) of the questions below directly on the test paper.

Name____________________

1. Recall that a family of subsets $\mathcal{F}$ of Ω is a $\pi-$system if, whenever $A_k \in \mathcal{F}$ for $k = 1, 2$ then $A_1 \cap A_2 \in \mathcal{F}$. A Boolean algebra of subsets is closed under finite intersections, finite unions and complementation.

   (a) Give an example of a family of subsets of the set $\{1, 2, 3\}$ that is a $\pi-$system but NOT a Boolean algebra of sets.

   (b) Give an example of a family of subsets that is a Boolean Algebra. Is this family also a Sigma Algebra?

   (c) Define a random variable on this space by $X(\omega) = 0$ if ω is odd, and otherwise $X(\omega) = 1$. What is $\sigma(X)$?

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2. *Indicate true or false and prove or find a counterexample:*

   (a) There exist two **independent** random variables X, Y defined on the space $\Omega = \{1, 2, 3, 4\}$ which have exactly the same distribution.

   (b) For independent random variables X, Y: it is always true that $\sigma(X, Y) = \sigma(X) \cup \sigma(Y)$.

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3. *Indicate true or false and prove or find a counterexample:*

   (a) For any non-negative random variable X, there is a sequence of simple random variables X_n such that $X_n(\omega) \leq X(\omega)$ for all $\omega \in \Omega$. and $E(X_n) \to E(X)$ as $n \to \infty$.

   (b) For **any integrable** random variable X (taking positive and negative values), there is a sequence of simple random variables X_n such that $X_n \to X$ a.s. and $E(X_n) \to E(X)$ as $n \to \infty$.

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4. *Indicate true or false and prove or find a counterexample:*

   (a) If X, Y are independent, nonegative random variables, $E(XY) = E(X)E(Y)$.

   (b) If two indicator random variables X, Y satisfy $E(XY) = E(X)E(Y)$ then they are independent.

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5. *Indicate true or false and prove or find a counterexample:*

(a) If
$$E(|X_n - X|) \to 0$$
as $n \to \infty$, then $X_n \to X$ in probability.

(b) If $P(\{\omega; X_n(\omega) \to X(\omega)\}) = 1$ for some integrable random variable X then $E(|X_n - X|) \to 0$.

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6. *Indicate true or false and prove or find a counterexample:*

(a) If X_n converges in probability to a random variable X then it converges in distribution (i.e. weakly) to X.

(b) If X_n converges in distribution to a random variable X then it converges in probability to X.