

Some Offbeat Geometry Problems

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1. Little Red Riding Hood gets lost in the woods. Within the woods she can travel at a maximum speed of 6 kilometers per hour. At 11 pm, the ghost of Archimedes appears to her and tells her that she must get out of the woods by midnight. After that she would be killed by the werewolves that infest the woods. She has the following information about the woods:

- The woods are about 2.8 square kilometers in area;
- The boundary of the woods is a simple closed curve.

What should Little Red Riding Hood do?

2. Same problem as before, except that the information is:
 - The woods are 5.7 square kilometers in area;
 - The woods are convex in shape.
3. Little Red is lost in a wooded area shaped like the upper half plane $\{(x, y) : y \geq 0\}$. She doesn't know where she is, except that Archimedes tells her that she is exactly one kilometer from the edge of the woods. Show that she can get out by walking not more than 6.4 kilometers.
4. Is it possible to cut a thin hole in the plane so that the plane remains connected, and so that a wire model of a cube of edge 1, say, can be pushed through the hole? The thickness of the wire is assumed to be negligible, and the area of the hole is assumed to be negligible as well.
5. Same problem, except that the wire model is a regular tetrahedron.
6. Does there exist a "space pentagon" whose five sides are equal (equilateral) and whose five angles are equal (equiangular), which does *not* lie in a plane?
7. Is it possible to shield a point source of light with four metal spheres?
8. A convex planar region has width one in every direction. Equivalently, the region casts shadows which are unit intervals on any line when illuminated from infinity perpendicular to the line. Is the region necessarily a circular disc?
9. Is it possible to partition three-dimensional space up into disjoint skew lines? That is, no two lines are parallel, no two intersect, and the union of all lines is all of 3-space.
10. Is it possible to partition three-dimensional space into loops. A loop is a simple closed curve in three dimensional space—like a one-dimensional elastic band.
11. True or false? There exists a polyhedron whose planar sections are all triangular.
12. True or false? If each vertex of a convex polyhedron is joined to every other vertex by edges, then it is a tetrahedron.
13. Prove that a convex polyhedron cannot have exactly seven edges.
14. A convex n -gon is triangulated by non-intersecting diagonals such that an odd number of triangles meets at any vertex. Prove that n is divisible by 3.