# University of Waterloo, Department of Statistics and Actuarial Science

Joint Rutgers-CMU-Waterloo PhD Topic Course

### STAT946/ACTSC991 Winter 2021

Game-theoretic statistical inference: betting, e-values and martingales

#### Time

Jan 22 – Apr 30 (Optional after Apr 9 for Waterloo students) Fridays, 10am - 1pm EST (with short break 11:45-12:00)

#### Instructor

Ruodu Wang (http://sas.uwaterloo.ca/~wang/) Email: wang@uwaterloo.ca

## **Description**

This course is a joint Rutgers-CMU-Waterloo PhD topic course which will be taught online (via Zoom) by Ruodu Wang (University of Waterloo), Glenn Shafer (Rutgers University) and Aaditya Ramdas (Carnegie Mellon University). We will study the game-theoretic foundation for statistics and its applications to prediction, testing and estimation. It will distinctively employ modern concepts around betting, e-values, and martingales, effectively bridging Bayesian, frequentist and model-free or adversarial perspectives on these topics. The course is based on very recent research results in a rapidly developing field.

#### Course website

https://www.stat.cmu.edu/~aramdas/betting/b21.html

The course website contains the course schedule and download links of all reading materials.

### **Prerequisites**

Basic graduate level (or advanced undergraduate) training in statistics or probability.

#### Office hours

Remotely by appointment

#### **Addition information**

- Glenn Shafer (<a href="http://www.glennshafer.com/">http://www.glennshafer.com/</a>), University Professor, Rutgers Business School, Rutgers University
- Aaditya Ramdas (<a href="http://stat.cmu.edu/~aramdas/">http://stat.cmu.edu/~aramdas/</a>), Assistant Professor, Department of Statistics and Data Science, Machine Learning Department, Carnegie Mellon University
- Ruodu Wang (<a href="http://sas.uwaterloo.ca/~wang/">http://sas.uwaterloo.ca/~wang/</a>), University Research Chair and Associate Professor, Department of Statistics and Actuarial Science, University of Waterloo

### Zoom link

The zoom link will be available later.

## Office hours (Waterloo students only)

Every Wednesday 10am to 11am (EST) I will host office hours on WebEx (the weeks of Jan 25 – April 5). The information can be found on Learn.

Basis of grading Homework 20%

Class participation 30% (evaluated via questions from pre-class reading)

Research project and oral presentation 50%

#### **Detailed schedule**

Week 1. January 22. Glenn Shafer.

Testing by betting. RSS paper [S1] with discussion and response

Week 2. January 29. Glenn Shafer.

Classical statistics with betting protocols. [SV Chapter 10, S2]

Week 3. February 5. Ruodu Wang.

E-values vs p-values: calibration, combination, and closed testing [VW1]

Week 4. February 12. Aaditya Ramdas.

Martingales, likelihood ratios, e-values and betting scores [RRLK, WRB]

Week 5. February 19. Aaditya Ramdas.

Composite, nonparametric null testing using nonnegative supermartingales [HRMS1] (Reading Week in Waterloo. Waterloo students can watch the video later if they choose not to attend the lecture this week.)

Week 6. February 26. Ruodu Wang.

False discovery rate control with e-values [WR1]

Week 7. March 5. Glenn Shafer.

Upper and lower expectations. Chapter 6, Shafer and Vovk [SV]

Week 8. March 12. Glenn Shafer.

Abstract theory of testing. Chapters 7 and 8, Shafer and Vovk [SV]

Week 9. March 19. (Ruodu Wang).

Spring Break in the US. RW hosts presentations for Waterloo students in this week.

Week 10. March 26. Ruodu Wang.

Discovery matrices with e-values [VW2, VW3]

Week 11. April 2. Aaditya Ramdas.

Nonparametric estimation via confidence sequences [HRMS2, WR2, WR3]

Week 12. April 9. Aaditya Ramdas.

Universality of martingales in "safe, anytime-valid testing/estimation" [RRLK, WRB]

(The following weeks will be optional for Waterloo students; they are encouraged to attend these lectures, although their grades do not depend on lectures after April 9.)

Week 13. April 16. Ruodu Wang.

P\*-values and randomized tests: A bridge between p-values and e-values [W]

Week 14. April 23. To be decided (possibly a guest lecturer)

Week 15. April 30. Student presentations and high-level discussion of the course

### Reading materials (see the course website for updates and links)

[HRMS1] Time-uniform Chernoff bounds via nonnegative supermartingales.

S. Howard, A. Ramdas, J. Sekhon, J. McAuliffe Prob. Surveys, 2020 <u>arxiv</u> <u>proc</u>

[HRMS2] Time-uniform, nonparametric, nonasymptotic confidence sequences.

S. Howard, A. Ramdas, J. Sekhon, J. McAuliffe Annals of Stat., 2021 arxiv

[RRLK] Admissible anytime-valid sequential inference must rely on nonnegative martingales. A. Ramdas, J. Ruf, M. Larsson, W. Koolen ArXiv, 2020 <u>arxiv</u>

- [S1] <u>Testing by betting: A strategy for statistical and scientific communication, with discussion and response.</u>
- G. Shafer. Journal of the Royal Statistical Society, Series A., 2020+
- [S2] How the game-theoretic foundation for probability resolves the Bayesian vs. frequentist standoff. G. Shafer. preprint, 2020 <u>preprint</u>
- [SV] Game-Theoretic Foundations for Probability and Finance.

G. Shafer and V. Vovk, Wiley 2019

[VW1] E-values: Calibration, combination, and applications.

V. Vovk, R. Wang. Annals of Stat., 2020+, arxiv

[VW2] True and false discoveries with e-values.

V. Vovk, R. Wang. ArXiv, 2020, arxiv

[VW3] True and false discoveries with independent e-values.

V. Vovk, R. Wang. ArXiv, 2020, arxiv

[W] Testing with p\*-values: Between p-values and e-values.

R. Wang. ArXiv, 2020, arxiv

[WR1] False discovery rate control with e-values.

R. Wang, A. Ramdas. ArXiv, 2020, arxiv

[WR2] Confidence sequences for sampling without replacement.

I. Waudby-Smith, A. Ramdas. Neural Information Processing Systems, 2020. arxiv

[WR3] Variance-adaptive confidence sequences by betting.

I. Waudby-Smith, A. Ramdas. ArXiv, 2020, arxiv

[WRS] Universal inference.

L. Wasserman, A. Ramdas, S. Balakrishnan. PNAS, 2020, arxiv proc