How Superadditive Can a Risk Measure Be?

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Outline

Basel Documents

From Basel Committee on Banking Supervision:

R1: Consultative Document, May 2012,
Fundamental review of the trading book

R2: Consultative Document, October 2013,
Fundamental review of the trading book: A revised market risk framework.

Basel Question

R1, Page 41, Question 8:

"What are the likely constraints with moving from VaR to ES, including any challenges in delivering robust backtesting, and how might these be best overcome?"

- Cont, Deguest and Scandolo (2010): ES is not robust, while VaR is.
- Gneiting (2011): ES is not elicitable, while VaR is.

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A feast for financial mathematicians and financial statisticians!
Review paper: Embrechts et al (2014).

more details

VaR and ES

In this talk, $X > 0$ is interpreted as a loss.

Definition

$\mathrm{VaR}_p(X)$, for $p \in (0,1)$,

$$\mathrm{VaR}_p(X) = F_X^{-1}(p) = \inf\{x \in \mathbb{R} : F_X(x) \geq \alpha\}.$$

Definition

$\mathrm{ES}_p(X)$, for $p \in (0,1)$,

$$\mathrm{ES}_p(X) = \frac{1}{1-p}\int_p^1 \mathrm{VaR}_\delta(X)\mathrm{d}\delta \underset{(F\ \text{cont.})}{=} \mathbb{E}\left[X | X > \mathrm{VaR}_p(X)\right].$$

VaR versus ES: Summary

Value-at-Risk

1. Always exists
2. Only frequency
3. Non-coherent risk measure (diversification problem)
4. Backtesting straightforward
5. Estimation: far in the tail
6. Model uncertainty: sensitive to dependence
7. (Almost) robust with respect to weak topology

Expected Shortfall

1. Needs first moment
2. Frequency and severity
3. Coherent risk measure (diversification benefit)
4. Backtesting an issue (non-elicitability)
5. Estimation: data limitation
6. Model uncertainty: sensitive to tail modeling
7. Robust with respect to Wasserstein distance

The Holy Triangle of Risk Measures

A risk measure $\rho : \mathcal{X} \to [-\infty, \infty]$. $\mathcal{X} \supset L^\infty$ is a set which is closed under addition and scaler multiplication.

There are many desired properties of a good risk measure. Some properties are without debate:

- cash-invariance: $\rho(X + c) = \rho(X) + c$, $c \in \mathbb{R}$;
- monotonicity: $\rho(X) \le \rho(Y)$ if $X \le Y$;
- zero-normalization: $\rho(0) = 0$;
- law-invariance: $\rho(X) = \rho(Y)$ if $X =_d Y$.

(A standard risk measure; those properties are not restrictive)

Another one is listed here as debatable:

- positive homogeneity: $\rho(\lambda X) = \lambda \rho(X)$, $\lambda \ge 0$.

The Holy Triangle of Risk Measures

In my opinion, in addition to being standard, the three key elements of being a good risk measure are

(C) Coherence (subadditivity): $\rho(X+Y) \leq \rho(X) + \rho(Y)$.
[aggregate regulation/capturing the tail/capital allocation/convex optimization]

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(C) Coherence (subadditivity): $\rho(X+Y) \leq \rho(X)+\rho(Y)$. [aggregate regulation/capturing the tail/capital allocation/convex optimization]

(A) Comonotonic additivity: $\rho(X+Y) = \rho(X)+\rho(Y)$ if X and Y are comonotonic. [economical interpretation/distortion representation/non-diversification identity]

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(A) Comonotonic additivity: $\rho(X+Y) = \rho(X) + \rho(Y)$ if X and Y are comonotonic. [economical interpretation/distortion representation/non-diversification identity]

(E) Elicitability [statistical advantage/backtesting straightforward].

The War of the Two Kingdoms

- Financial mathematicians
  - appreciate coherence (subadditivity);
  - favor ES in general.
- Financial statisticians
  - appreciate backtesting and statistical advantages;
  - favor VaR in general.

A natural question is to find a standard risk measure which is both coherent (subadditive) and elicitable.

Expectiles

Expectiles

For $0 < p < 1$ and $X \in L^2$, the p-expectile is

$$e_p(X) = \operatorname*{argmin}_{x \in \mathbb{R}} \mathbb{E}[p(X-x)_+^2 + (1-p)(x-X)_+^2].$$

- $e_p(X)$ is the unique solution x of the equation for $X \in L^1$:

$$p\mathbb{E}[(X-x)_+] = (1-p)\mathbb{E}[(x-X)_+].$$

- $e_{1/2}(X) = \mathbb{E}[X]$.
- If we allow $p = 1$: $e_1(X) = \text{ess-sup}(X)$.

Expectiles

The risk measure e_p has the following properties:

1. positive homogeneous and standard;
2. subadditive for $1/2 \le p < 1$, superadditive for $0 < p \le 1/2$;
3. elicitable;
4. coherent for $1/2 \le p < 1$;
5. not comonotonic additive in general.

Bellini et al. (2014), Ziegel (2014), Delbaen (2014).

The War of the Three Kingdoms

In summary:

- VaR has (A) and (E): often criticized for not being subadditive: diversification/aggregation problems and inability to capture the tail!

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The War of the Three Kingdoms

In summary:

- VaR has (A) and (E): often criticized for not being subadditive: diversification/aggregation problems and inability to capture the tail!
- ES has (C) and (A): criticized for estimation, backtesting and robustness problems!
- Expectile has (C) and (E): criticized for lack of economical meaning, difficulty to conceptualize, distributional computation and over-diversification benefits!

The War of the Three Kingdoms

The following hold:

- if ρ is coherent, comonotonic additive and elicitable, then ρ is the mean (Ziegel, 2014);
- if ρ is coherent, and elicitable with a convex scoring function, then ρ is an expectile (Bellini and Bignozzi, 2014);
- if ρ is comonotonic additive, and elicitable, then ρ is a VaR or the mean (Kou and Peng, 2014, W. and Ziegel, 2014).

The War of the Three Kingdoms

In summary:

The only standard risk measure that has (C), (A) and (E) is the mean, which is not a tail risk measure, and does not have a risk loading.

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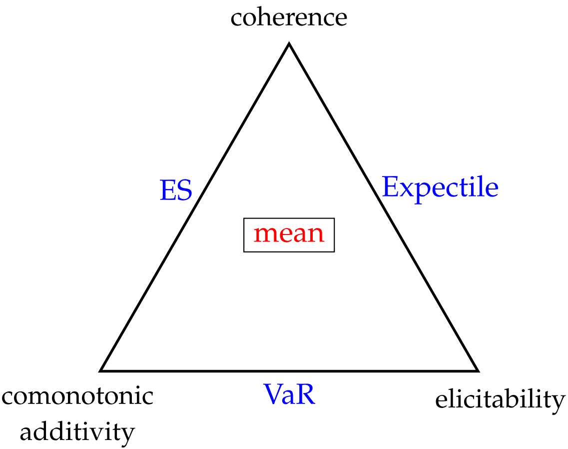
The only standard risk measure that has (C), (A) and (E) is the mean, which is not a tail risk measure, and does not have a risk loading.

- Remark: the very old-school risk measure/pricing principle $\rho(X) = (1+\theta)\mathbb{E}[X]$, $\theta > 0$ has (C-subadditivity), (A) and (E), although it is not standard.

The Holy Triangle of Risk Measures



Subadditivity

Subadditivity has to do with

- diversification benefit - *"a merger does not create extra risk"*.
- aggregation - manipulation of risk: $X \to Y + Z$;
- capturing the tail;
- convex optimization and capital allocation.

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- capturing the tail;
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It is questioned from different aspects:

- aggregation penalty - convex risk measures;
- robustness and backtesting;
- financial practice - *"a merger creates extra risk"*.

How Superadditive Can a Risk Measure be?

Question: given an non-subadditive risk measure,

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How superadditive can it be?

Motivation:

- Measure model uncertainty.
- Quantify worst-scenarios.
- Trade subadditivity for statistical advantages such as robustness or elicitability.
- Understand better about subadditivity.

Diversification ratio

For a law-invariant (always assumed) risk measure ρ, and risks $\mathbf{X} = (X_1, \cdots, X_n)$, the diversification ratio is defined as

$$\Delta^{\mathbf{X}}(\rho) = \frac{\rho(X_1 + \cdots + X_n)}{\rho(X_1) + \cdots + \rho(X_n)}.$$

For the moment, the denominator is assumed positive.

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For the moment, the denominator is assumed positive.

- $\Delta^{\mathbf{X}}(\rho)$ is important in modeling portfolios.
- We want to know how large $\Delta^{\mathbf{X}}(\rho)$ can be.
- $\Delta^{\mathbf{X}}(\rho) \leq 1$ for subadditive risk measures.
- We cannot take a supremum over all possible $\mathbf{X}$, which often explodes for any non-superadditive risk measure.

Diversification ratio

We define a law-invariant version of the diversification ratio:

$$\Delta_n^F(\rho) = \sup\left\{\frac{\rho(X_1+\cdots+X_n)}{\rho(X_1)+\cdots+\rho(X_n)} : X_1,\ldots,X_n \sim F\right\}.$$

Here we assumed homogeneity in F_i for:

- mathematical tractability;
- that it makes sense to let n vary;
- that it also catches superadditivity of ρ for inhomogeneous portfolio.

Diversification ratio

Define

$$\mathfrak{S}_n(F) = \{X_1 + \cdots + X_n : X_1, \ldots, X_n \sim F\}.$$

Let $X_F \sim F$. Then

$$\Delta_n^F(\rho) = \frac{1}{n\rho(X_F)} \sup \{\rho(S) : S \in \mathfrak{S}_n(F)\}.$$

- Known to be a difficult problem; explicit solution for $\Delta_n^F(\mathrm{VaR}_p)$ (under some strong conditions) given in W., Peng and Yang (2013).
- Numerical calculation for $\Delta_n^F(\mathrm{VaR}_p)$ given in Embrechts, Puccetti and Rüschendorf (2013).

Extreme-aggregation measure

We are interested in the global superadditivity ratio

$$\Delta^F(\rho) = \sup_{n\in\mathbb{N}} \Delta_n^F(\rho) = \sup_{n\in\mathbb{N}} \frac{1}{n\rho(X_F)} \sup\left\{\rho(S) : S \in \mathfrak{S}_n(F)\right\}.$$

$\Delta^F(\rho)$ characterizes how superadditive ρ can be for a fixed F.

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$\Delta^F(\rho)$ characterizes how superadditive ρ can be for a fixed F.

The real mathematical target:

$$\sup_{n\in\mathbb{N}} \frac{1}{n} \sup\left\{\rho(S) : S \in \mathfrak{S}_n(F)\right\}.$$

A closely related quantity:

$$\limsup_{n\to\infty} \frac{1}{n} \sup\left\{\rho(S) : S \in \mathfrak{S}_n(F)\right\}.$$

Extreme-aggregation measure

Definition

An extreme-aggregation measure induced by a law-invariant risk measure ρ is defined as

$$\Gamma_\rho : \mathcal{X} \to [-\infty, \infty], \quad \Gamma_\rho(X_F) = \limsup_{n\to\infty} \frac{1}{n} \sup\left\{\rho(S) : S \in \mathfrak{S}_n(F)\right\}.$$

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- Γ_ρ quantifies the limit of ρ for worst-case aggregation under dependence uncertainty.
- Γ_ρ is a law-invariant risk measure.

Extreme-aggregation measure

Proposition

If ρ is (i) comonotonic additive, or (ii) convex and zero-normalized, then

$$\Gamma_\rho(X_F) = \sup_{n \in \mathbb{N}} \frac{1}{n} \sup \{\rho(S) : S \in \mathfrak{S}_n(F)\} \geq \rho(X_F)$$

If ρ is subadditive then $\Gamma_\rho \leq \rho$. If it also satisfies (i), or (ii), then $\Gamma_\rho = \rho$.

Extreme-aggregation measure

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Remark

Γ_ρ inherits monotonicity, cash-invariance, positive homogeneity, subadditivity, convexity, or zero-normalization from ρ if ρ has the corresponding properties.

Extreme-aggregation measure

Question: given a non-subadditive risk measure ρ,

What is Γ_ρ?

Extreme-aggregation measure

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What is Γ_ρ?

- Known motivating result (Wang and W., 2014): as $n \to \infty$,

$$\frac{\sup\{\mathrm{VaR}_p(S) : S \in \mathfrak{S}_n(F)\}}{\sup\{\mathrm{ES}_p(S) : S \in \mathfrak{S}_n(F)\}} \to 1.$$

Note that

$$\sup\{\mathrm{ES}_p(S) : S \in \mathfrak{S}_n(F)\} = n\mathrm{ES}_p(X_F),$$

leading to $\Gamma_{\mathrm{VaR}_p} = \Gamma_{\mathrm{ES}_p} = \mathrm{ES}_p$.

Distortion risk measures

Distortion risk measures:

$$\rho(X_F) = \int_0^1 F^{-1}(t)\mathrm{d}h(t).$$

h: probability measure on $(0, 1)$. A distortion function.

Distortion risk measures

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$$\rho(X_F) = \int_0^1 F^{-1}(t)\mathrm{d}h(t).$$

h: probability measure on $(0, 1)$. A distortion function.

- We assume random variables are bounded from below: $F^{-1}(0) > -\infty$.
- Standard risk measures are comonotonic additive if and only if it is a distortion risk measure (a property of Choquet integral; see Yaari, 1987).
- ES, VaR are special cases of distortion risk measures.

Distortion risk measures

Theorem (Extreme-aggregation for distortion risk measures)

Suppose ρ is a distortion risk measure with distortion function h, then Γ_ρ is

(a) *the smallest coherent risk measure dominating ρ;*

(b) *a coherent distortion risk measure with a distortion function as the largest convex distortion function dominated by h.*

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- Example: $\Gamma_{\mathrm{VaR}_p} = \mathrm{ES}_p$.
- A proof quite complicated.

Main result for distortion risk measures

For distortion risk measures,

$$\Delta^F(\rho) = \frac{\Gamma_\rho(X_F)}{\rho(X_F)}.$$

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Theorem (Coherence and extreme-aggregation)

Suppose ρ is distortion risk measure. The following are equivalent:

(a) *ρ is coherent.*

(b) *$\Gamma_\rho(X_F) = \rho(X_F)$ for all distributions F.*

(c) *$\Gamma_\rho(X_F) = \rho(X_F)$ for some continuous distribution F, $\rho(X_F) < \infty$.*

(d) *$\Delta^F(\rho) = 1$ for all distributions F, $\rho(X_F) \in (0, \infty)$.*

(e) *$\Delta^F(\rho) = 1$ for some continuous distribution F, $\rho(X_F) \in (0, \infty)$.*

Shortfall risk measures

Shortfall risk measures:

$$\rho(X) = \inf\{y \in \mathbb{R} : \mathbb{E}[\ell(X - y)] \leq l(0)\}.$$

ℓ: convex and increasing function. A loss function.

- Motivation from indifference pricing theory.

Shortfall risk measures

Theorem (Extreme-aggregation for shortfall risk measures)

Suppose ρ is a shortfall risk measure with loss function ℓ, then Γ_ρ is

(a) *the smallest coherent risk measure dominating ρ;*

(b) *a coherent p-expectile, where*

$$p = \lim_{x\to\infty} \ell'(x)/(\lim_{x\to\infty} \ell'(x) + \lim_{x\to-\infty} \ell'(x))$$

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- Example: $\Gamma_{\mathrm{ER}_\beta} = e_1 = \text{ess-sup}$, where ER_β is the entropy risk measure: with loss function $\ell(x) = \exp(\beta x) - 1$.
- A proof of one page.

Convex risk measures

A convex risk measure is standard and convex:

$$\rho(\lambda X + (1-\lambda)Y) \leq \lambda\rho(X) + (1-\lambda)\rho(Y), \quad \lambda \in (0,1).$$

Theorem (Extreme-aggregation for convex risk measures)

Suppose ρ is a convex risk measure, then Γ_ρ is a coherent risk measure.

Convex risk measures

A law-invariant convex risk measure has the following representation

$$\rho = \sup_{h \in \mathcal{P}[0,1]} \left\{ \int \mathrm{ES}_p \mathrm{d}h(p) - v(h) \right\},$$

where

- $\mathcal{P}[0,1]$ is the set of all probability measures on $[0,1]$;
- $v : \mathcal{P}[0,1] \to \mathbb{R} \cup \{+\infty\}$ is a convex function;
- $\rho(0) = 0$ is equivalent to $\inf_{h \in \mathcal{P}[0,1]} v(h) = 0$.

Extreme-aggregation for convex risk measures

Extreme-aggregation for convex risk measures

Γ_ρ is a coherent risk measure with representation

$$\Gamma_\rho = \sup_{h \in \mathcal{Q}} \left\{ \int \mathrm{ES}_p \mathrm{d}h(p) \right\},$$

where $\mathcal{Q} = \{h \in \mathcal{P}[0,1] : v(h) > -\infty\}$.

- Γ_ρ is the smallest coherent risk measure dominating ρ.

Extreme-division

We define the extreme-division measure (W., 2014) for a risk measure ρ:

$$\Psi_\rho(X) = \inf\left\{\sum_{i=1}^n \rho(X_i) : n \in \mathbb{N},\ X_i \in \mathcal{X}, i = 1, \dots, n,\ \sum_{i=1}^n X_i = X\right\}.$$

- $\Psi_\rho(X)$ is the least amount of capital requirement according to ρ if the risk X can be divided arbitrarily.
- $\Psi_\rho \leq \rho$.
- $\Psi_\rho = \rho$ for subadditive risk measures.
- Not relevant to this topic, just wanted to show an interesting duality.

Extreme-division for convex risk measures

Extreme-division for convex risk measures

Ψ_ρ is a coherent risk measure with representation

$$\Psi_\rho = \sup_{h \in \mathcal{Q}} \left\{ \int \mathrm{ES}_p \mathrm{d}h(p) \right\},$$

where $\mathcal{Q} = \{h \in \mathcal{P}[0,1] : v(h) = 0\}$.

- Ψ_ρ is the largest coherent risk measure dominated by ρ.
- When ρ is a distortion risk measure, Ψ_ρ is a coherent risk measure, but not necessarily a distortion.
- $\Psi_{\mathrm{VaR}_p} = -\infty$ for all $p \in (0,1)$.

Discussion

- Γ_ρ is very often a coherent risk measure for all commonly used standard risk measures. However, counter-example can be built.
- Γ_ρ often gains positive homogeneity, convexity, and subadditivity even if ρ does not have these properties.
- A universal axiomatic proof of this phenomenon is not available yet.
- Characterize the class of risk measures which induce coherent extreme-aggregation measures?
- What happens to shortfall risk measures with non-convex loss functions?

Discussion

Some take-home message:

Coherence is indeed a natural property desired by a good risk measure. Even when a non-coherent risk measure is applied to a portfolio, its extreme behavior under dependence uncertainty leads to coherence.

When we allow arbitrary division of a risk, the extreme behavior also leads to coherence.

This contributes to the Basel question and partly supports the use of coherent risk measures.

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Thank you for your kind attendance!

Basel Question

R1, Page 20, *Choice of risk metric*:

"... However, a number of weaknesses have been identified with VaR, including its inability to capture "tail risk". The Committee therefore believes it is necessary to consider alternative risk metrics that may overcome these weaknesses."

Basel Question

We focus on the mathematical and statistical aspects, avoiding discussion on practicalities and operational issues.

R1, Page 3:

"The Committee recognises that moving to ES could entail certain operational challenges; nonetheless it believes that these are outweighed by the benefits of replacing VaR with a measure that better captures tail risk."

Basel Question

R2, Page 3, *Approach to risk management*:

"the Committee has its intention to pursue two key confirmed reforms outlined in the first consultative paper [May 2012]: Stressed calibration . . . Move from Value-at-Risk (VaR) to Expected Shortfall (ES)."

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